1962

An analysis of a divergent model for numerical forecasting.

Nicholson, Harry E.

Monterey, California. Naval Postgraduate School

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HARRY E. NICHOLSON

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MODEL FOR NUMERICAL FORECASTING

by

Harry E. Nicholson

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

A forecast model proposed by 'Arnason as being capable of producing realistic forecasts of displacement of systems at and below 500 mb is investigated. The prediction equation,

$$\frac{\partial \zeta}{\partial t} + \mathbb{V} \cdot \nabla \eta - \eta \left[\frac{4\bar{F}}{g H_s} \left(\frac{\partial \psi}{\partial t} + k_1 \overline{\mathbb{V}} \cdot \nabla \psi \right) \right] = 0$$

is applied to test cases at 1000, 850, 700 and 500 mb with various values of the parameter k_1. The effect of the value of the parameter on the amount of divergence present in the model is discussed. Root-mean-square errors for 24-hour and 48-hour forecasts with various values of k_1 at each level are presented. Optimum values of k_1 for the cases tested are selected as 3.5, 2.75 and 0.9 for the levels 850, 700 and 500 mb respectively. It is found that with proper values for k_1 the model will provide good forecasts of system movement at each level tested, but as expected, changes in the intensity of systems are not correctly forecast. In all cases values of root-mean-square error of forecasts are less than those obtained by applying the "Helmholtz" barotropic model. A geostrophic version of the model is also tested and is found to produce results similar to those obtained for the stream-function version except that the geostrophic "blow-up" of low-latitude high pressure cells is observed.

The writer wishes to express his appreciation for the guidance and encouragement given him by Professor George J. Haltiner of the U. S. Naval Postgraduate School in the course of this investigation. In addition, appreciation is expressed to Mr. Geirmundur 'Arnason, on whose work this paper is based, and to personnel of the Fleet Numerical Weather Facility for many instances of assistance.

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TABLE OF SYMBOLS

$\mathbb{V}$ - the horizontal vector wind

$\mathrm{div}\,\mathbb{V}$ - the divergence of the horizontal wind

ϕ - the geopotential

σ' - a parameter of the stratified model related to the stability of the basic flow

H - the thickness of the model atmosphere

$\overline{\mathbb{V}}$ - the horizontal vector mean wind

η - the absolute vorticity

ζ - the relative vorticity

ψ - the non-divergent stream function

f - the Coriolis parameter

$\bar{f}$ - the value of the Coriolis parameter at 45N

g - the acceleration of gravity

c - phase velocity

U - speed of the basic current

k - number of waves in the x-direction

H_5 - the standard height of the 500-mb surface

β - $\frac{df}{dy}$

1. Introduction

Since the initation of operational numerical weather forecasts by the Joint Numerical Weather Prediction Unit in 1955, the only forecast model which has stood the test of daily operational use has been the barotropic model first proposed by Rossby in 1939. The barotropic model remained in its original form until 1958 when Wolff [1] discovered large systematic errors due to improper forecast of very long waves. After these errors had been empirically corrected by Wolff, it was discovered by Cressman [2] that these long-wave errors could be reduced by the introduction of divergence into the barotropic model. This resulted in the divergent or "Helmholtz" version of the barotropic model which is in general use at this time.

With the improved barotropic model, high quality forecasts have been available for the 500-mb level. This barotropic model, however, is by nature restricted in its use to the 500-mb level and, in addition, is capable mainly of forecasting the movement of systems, and not in general changes in intensity. Occasionally attempts have been made to replace the barotropic model in operational use with some type of baroclinic model which would obviate these shortcomings. These attempts have so far met with failure largely due to the inability to control the forecast of cyclogenesis.

'Arnason has suggested that, in view of the great utility of the 500-mb forecasts produced by the barotropic model, it might be worthwhile temporarily to set aside the problem of predicting cyclogenesis and to try to develop a model which would possess the same skill of forecasting movement at other levels which the barotropic has at 500 mb. To this end, 'Arnason has proposed a model [3] which maintains many of the

characteristics of the barotropic, but which is more adaptable to other levels.

This model is based on a single-layer compressible fluid with a stratified density distribution, bounded by two constant-pressure surfaces. The theoretical development results in an expression for the horizontal wind divergence

$$\operatorname{div} \mathbb{V} = \frac{1}{g' H}\left(\frac{\partial \phi}{\partial t} + \mathbb{V} \cdot \nabla \phi\right). \tag{1}$$

Introducing this expression into the vorticity equation,

$$\frac{\partial \zeta}{\partial t} + \mathbb{V} \cdot \nabla \eta - \eta \operatorname{div} \mathbb{V} = 0, \tag{2}$$

gives the prediction equation for the "stratified" model

$$\frac{\partial \zeta}{\partial t} + \mathbb{V} \cdot \nabla \eta - \frac{\eta}{g' H}\left(\frac{\partial \phi}{\partial t} + \overline{\mathbb{V}} \cdot \nabla \phi\right) = 0. \tag{3}$$

In order to apply this equation to forecasting, 'Arnason has suggested, as one alternative, the use of a stream function for computation of wind and vorticity and the elimination of ϕ through a geostrophic assumption. This gives

$$\left(\nabla^2 - \frac{f\eta}{g' H}\right)\frac{\partial \psi}{\partial t} + \mathbb{V} \cdot \nabla \eta - \frac{f\eta}{g' H}\, \overline{\mathbb{V}} \cdot \nabla \psi = 0, \tag{4}$$

which is of the same form as the prediction equation for the divergent or "Helmholtz" barotropic model, but with an additional divergence term involving $\overline{\mathbb{V}}$ and with the substitution of the stability-dependent term g' for g.

Although the forms of the prediction equations are similar, there are several basic differences between the "stratified" and other barotropic models. By the method of perturbation analysis 'Arnason has obtained

the meteorologically significant root of the frequency equation for the stratified model

$$C = U - \left(\frac{g'H}{g'H + f/k} \right) \beta/k^2 . \tag{5}$$

Although this equation is again similar to the corresponding equation for the Helmholtz barotropic model, an important difference arises in the fact that the basic current U does not have a multiplying factor. By empirically varying the value of g'H, the magnitude of the β term may be changed without reducing the effect of the basic current, thus allowing control over the speed of movement of systems. As can be seen from equation (1) this is equivalent to a variation in the amount of divergence present in the model. This provides a method of adjusting the model to yield proper system movement at a number of levels.

The stratified model as proposed by 'Arnason seems to provide a promising approach to numerical forecasting at and below the 500-mb level. It is the purpose of this paper to apply the model to a number of test cases and determine its effectiveness.

2. Application of the Model to Forecasting

In the application of the prediction equation (3) to actual forecasts there are several choices to be made. The parameter $\overline{\mathbb{V}}$ has not been explicitly defined, but may be taken to be any kind of space-averaged or zonal wind. Also the value of g'H is left to be determined empirically.

By examination of the prediction equation (3), it can be seen that the additional "divergence" term of the stratified model which involves $\overline{\mathbb{V}}$ essentially represents the horizontal advection of height by the mean wind. This immediately suggests the notion of a steering current, which may be thought of as a wind at some level or as a mean wind in some layer of the atmosphere. Also, the magnitude of the divergence term is dependent on the difference between the mean wind and the wind at the forecast level. Thus it appears reasonable to use as a mean wind something close to the 500-mb wind, since this would minimize the divergence term at 500 mb and allow it to increase with distance from 500 mb. This is generally consistent both with observed patterns of divergence in the atmosphere and with the need for increasing the value of the divergence term at lower levels to supplement the movement due to vorticity advection alone.

A second parameter which must be empirically interpreted is the factor g'H. The value of g' is a measure of the static stability of the basic flow and H is the thickness of the layer considered in the model. Since neither is uniquely defined, the combination may be taken as any reasonable value which produces proper movement of the systems being forecast.

In order to make a forecast, either ψ or ϕ must be eliminated

from equation (3). 'Arnason has used a geostrophic relation to eliminate ϕ, obtaining equation (4) which involves ψ alone. To apply this equation, height fields must first be "balanced" to obtain the stream function and then, at the conclusion of the forecast, inverted to obtain the forecast height field. Another approach would be to compute the vorticity and winds directly from the height values using the geostrophic relationship, obtaining the forecast equation:

$$\left(\nabla^2 - \frac{f\eta}{g'H}\right)\frac{\partial z}{\partial t} + \frac{f}{g}\mathbb{V}\cdot\nabla\eta - \frac{f\eta}{g'H}\overline{\mathbb{V}}\cdot\nabla z = 0 \tag{6}$$

3. Procedure

As mentioned above, in order to apply the stratified model to actual forecasting, values must be chosen for the parameters $\overline{\mathbb{V}}$ and g'H. For the purposes of the tests presented here, it was decided to use for $\overline{\mathbb{V}}$ a mean wind computed from a height field obtained by first heavily smoothing the 500-mb and 700-mb heights, then computing an intermediate field by the equation

$$\phi = .75\,\phi_{500} + .25\,\phi_{700}\,. \tag{7}$$

The wind obtained by this method is thus a space-mean wind for some intermediate level between 500 mb and 700 mb. For the first six one-hour time steps, a $\overline{\mathbb{V}}$ computed from the initial data was used; thereafter a new $\overline{\mathbb{V}}$ was computed from forecast fields every sixth time step.

In the choice of the parameter g'H, consideration was given to the similarity between the forecast equation (4) and the forecast equation for the "Helmholtz" barotropic

$$\left(\nabla^2 - \frac{k\bar{f}\eta}{gH_5}\right)\frac{\partial\psi}{\partial t} + \mathbb{V}\cdot\nabla\eta = 0\,. \tag{8}$$

Since it was known that forecasts made with this model at 500 mb are of consistently good quality, it was decided to choose g'H such that the Helmholtz term for the stratified model would be the same as the Helmholtz term of the barotropic. Thus in the Helmholtz term, g'H was chosen to be

$$g'H = \frac{gH_5f}{k\bar{f}} \tag{9}$$

where k is a "tuning constant" with a value of four, and H_5 is the standard height of the 500-mb surface.

In order that the model be used at levels other than 500 mb, it was

decided to include a second "tuning constant" in the final divergence term. By varying this second constant the proper amount of divergence may be introduced at each level to produce the best forecast. With these values introduced, the forecast equation for the stream function case becomes

$$\left(\nabla^2 - \frac{4\bar{f}\eta}{gH_s}\right)\frac{\partial \psi}{\partial t} + \mathbb{V}\cdot\nabla\eta - \frac{4k_1\bar{f}\eta}{gH_s}\bar{\mathbb{V}}\cdot\nabla\psi = 0 \tag{10}$$

In the same manner, starting with equation (6) a similar equation was obtained for the geostrophic version of the model:

$$\left(\nabla^2 - \frac{4\bar{f}\eta}{gH_s}\right)\frac{\partial z}{\partial t} + \frac{f}{g}\mathbb{V}\cdot\nabla\eta - \frac{4k_1\bar{f}\eta}{gH_s}\bar{\mathbb{V}}\cdot\nabla z = 0 \tag{11}$$

The model, in the form of finite difference equations obtained from equations (10) and (11), was run to 48-hour forecasts for 850 mb, 700 mb and 500 mb for several randomly selected days. Various values of k_1 were used at each level for both the geostrophic and stream-function cases. In addition, for the 6 May 1962 case runs were made on 1000-mb data in order to determine the feasibility of using the model at that level.

4. Computational Methods

The test computations of the stratified model described in this paper were made on a Control Data Corporation 1604 computer using a program constructed of meteorological subroutines prepared by the Fleet Numerical Weather Facility, Monterey. The grid used in the numerical solution of the prediction equation was the standard octagonal grid used by both the Joint Numerical Weather Prediction Unit at Suitland, Md. and the Fleet Numerical Weather Facility.

Time differencing was accomplished in one-hour time steps using the central difference formula

$$\phi_{\tau+1} = \phi_{\tau-1} + 2\left(\Delta_{*}\phi\right)_{\tau} , \tag{12}$$

where $\Delta_{*}\phi$ is the computed one-hour change in geopotential. At the initial time, $\tau = 0$, and all integral multiples of twelve, the forward difference formula

$$\phi_{\tau+1} = \phi_{\tau} + \left(\Delta_{*}\phi\right)_{\tau} \tag{13}$$

was used.

In cases where a stream function wind was used, the stream values were obtained from initial height data through the solution of the balance equation as described by 'Arnason [4]. All initial height fields and fields required for forecast verification were obtained from files of operational analyses at the Fleet Numerical Weather Facility.

5. Results

Verification scores for each of the cases tested were determined by a FNWF program using the equations

$$\text{Pillow} = \frac{\sum_{n=1}^{N} (A-B)_n}{N} \tag{14}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{n=1}^{N} \left[(A-B)_n - \text{Pillow}\right]^2}{N}} \tag{15}$$

Here A represents the forecast value, B, the observed value and N, the total number of grid points, 1977.

As can be seen from the equations, the pillow is a measure of the error in the forecast of the mean height while the RMSE (root mean square error) is a measure of the deviation of errors around the mean error.

Figures 1, 2 and 3 show plots of RMSE as a function of k_1 for the various cases tested. As can be seen, the value of k_1 necessary to produce the best forecast (minimum RMSE) varies somewhat from case to case at each level. However, the curves are relatively flat in the vicinity of the minima so that by selecting a reasonable value of k_1, the quality of forecasts from one case to another will vary only slightly from the optimum. Table 1 shows mean values of RMSE for all cases obtained with various values of k_1 at each level. A plot of these values is shown in figure 4. From this plot of mean error versus k_1, values of k_1 can be selected which will give the minimum total error. Table 2 gives a tabulation of the error which would occur in each case tested if k_1 at each level were selected as the value which produces the smallest mean error.

Forecasts for 850 mb made from initial data for 20 January 1962 are

Table 1. Mean Verification Scores of Forecasts With Various Values of k_1

STREAM-FUNCTION FORECASTS									
		k_1	RMSE (ft)	k_1	RMSE (ft)	k_1	RMSE (ft)	k_1	RMSE (ft)
24 hours	850	2	112	3	107	4	108	5	114
	700	1	133	2	124	3	123	4	128
	500	0	148	$\frac{1}{2}$	147	1	147	2	150
48 hours	850	2	174	3	166	4	166	5	173
	700	1	189	2	174	3	169	4	173
	500	0	219	$\frac{1}{2}$	215	1	214	2	218
GEOSTROPHIC FORECASTS									
		k_1	RMSE (ft)	k_1	RMSE (ft)	k_1	RMSE (ft)	k_1	RMSE (ft)
24 hours	850	2	130	3	123	4	125	5	134
	700	1	137	2	127	3	126	4	131
	500	0	163	$\frac{1}{2}$	161	1	162	2	169
48 hours	850	2	198	3	192	4	193	5	200
	700	1	---	2	203	3	195	4	198
	500	0	---	$\frac{1}{2}$	271	1	269	2	277

Table 2. Verification Scores For Each Day Using Optimum Value of k_1

	850 mb RMSE (ft) k_1=3.5	700 mb RMSE (ft) k_1=2.75	500 mb RMSE (ft) k_1=0.9
4 November 1961 (24 hr)	111	116	134
20 January 1962 (24 hr)	128	128	178
6 May 1962 (24 hr)	87	88	127
4 November 1961 (48 hr)	173	185	226
20 January 1962 (48 hr)	198	185	250
6 May 1962 (48 hr)	143	140	168

shown in figures 8 through 13. Initial charts and charts for the verifying time of each forecast are shown in figures 5 through 7. Comparing forecasts for the various values of k_1 at a given level shows that the forecast speed of synoptic waves increases as the value of k_1 is increased (thus decreasing the value of $g'H$ and increasing the magnitude of divergence). This is in accordance with the results of equation (5). Further, it can be seen that there is very little change in intensity of the various systems as k_1 is varied. This demonstrates the point that the additional divergence term of the stratified model results in movement rather than development.

As can be seen from equations (10) and (11) a choice of $k_1=0$ would reduce the forecast to the barotropic. An examination of figure 4 reveals that the optimum value of k_1 at 500 mb is close to zero, but that the stratified model does show a slight improvement over the barotropic with values of k_1 between zero and one. This is as expected, since 500 mb is generally accepted to be at or near the level of minimum divergence, and thus the contribution of divergence to the change in the vorticity pattern at this level should be small.

At levels other than 500 mb, where the barotropic model is less adequate, the values of k_1 for which the minimum RMSE is reached are greater. For the 700-mb level optimum values for the cases tested range from $k_1=2$ to $k_1=4$, while at 850 mb they range from 3 to 5. These results can be accounted for by the vertical distribution of velocity divergence which generally increases with distance from the 500-mb surface.

By use of equation (1) and the chosen values for $g'H$ given by equation (9), an expression for the divergence at a given time may be obtained:

$$\mathrm{div}\,\mathbb{V} = \frac{4\bar{f}}{g H_s f}\left(\frac{\partial \phi}{\partial t} + k_1 \overline{\mathbb{V}} \cdot \nabla \phi\right) \tag{16}$$

In figures 14, 15 and 16 divergence charts at 850, 700 and 500 mb as computed from equation (16) for initial data of 20 January 1962 are shown. These charts show clearly the vertical distribution of divergence, with values as large as $\pm$ 5 x 10^{-6} sec^{-1} at 850 mb, $\pm$ 3 x 10^{-6} sec^{-1} at 700 mb and values very nearly zero at 500 mb.

An examination of the divergence fields relative to their corresponding height fields reveals that lines of zero divergence very closely follow the short-wave trough and ridge lines of the corresponding height fields. Between the short-wave trough and ridge lines are located the centers of maximum convergence and divergence, with divergence ahead of ridges and convergence ahead of troughs, as might be expected.

For the case of 6 May 1962, 1000-mb forecasts were made. From figure 3 it can be seen that the curves of RMSE versus k_1 for 1000 mb are almost exactly parallel to the curves for 850 mb with an RMSE about ten feet greater at 1000 mb than at 850 mb. The fact that the curves are nearly parallel suggests that the increased error at 1000 mb is due to a greater change in the intensity of systems at this level than at 850 mb. The movement of established systems which appear on both initial charts is nearly the same at both levels; thus k_1 which controls the magnitude of the additional movement term should reasonably be chosen the same for the two levels. The verifying charts for the forecasts in question, figures 21, 22, 25 and 26, show this to be the case. Forecasts at both levels have the major systems placed very close to their actual positions at verifying time, but the verifying charts at 1000 mb differ

more in pattern due to the variation in intensity change of different systems over the forecast period.

In addition to the forecasts made for the three days using stream functions obtained from the initial height fields, for two of the days forecasts were made directly from the height fields using equation (11). As can be seen from figures 27 through 30, the geostrophic forecasts thus obtained showed great similarity to the forecasts made from the stream function. However, as expected, the geostrophic version appears to suffer from the major error common to geostrophic forecasts - the "blow-up" of low-latitude high-pressure areas. As can be seen, the 48-hour geostrophic forecast increases the central height of the 500-mb high off the east coast of the United States to over 19,500 feet, while the stream-function version keeps the central height to a more reasonable value. The treatment of the high off the west coast of the United States is similar.

6. Conclusions

The stratified model as proposed by 'Arnason appears, on the basis of the test cases presented here, to produce reasonable forecasts for levels at and below 500 mb. The stream-function version of the stratified model produces 500-mb forecasts for 24 and 48 hours which are in all cases very slightly better than those produced by the barotropic model. At 850 mb and 700 mb the model produces forecasts of system movement which are very realistic. The geostrophic version of the model suffers from the "blow-up" of low-latitude high pressure cells, as is common with most geostrophic models.

Optimum values of the multiplying factor of the divergence term k_1 at 850, 700 and 500 mb have been found to be 3.5, 2.75 and 0.9 respectively. Although the number of cases tested was not large, the values determined here should be sufficiently accurate to provide a basis for the long-term testing necessary before any model becomes suitable for operational use.

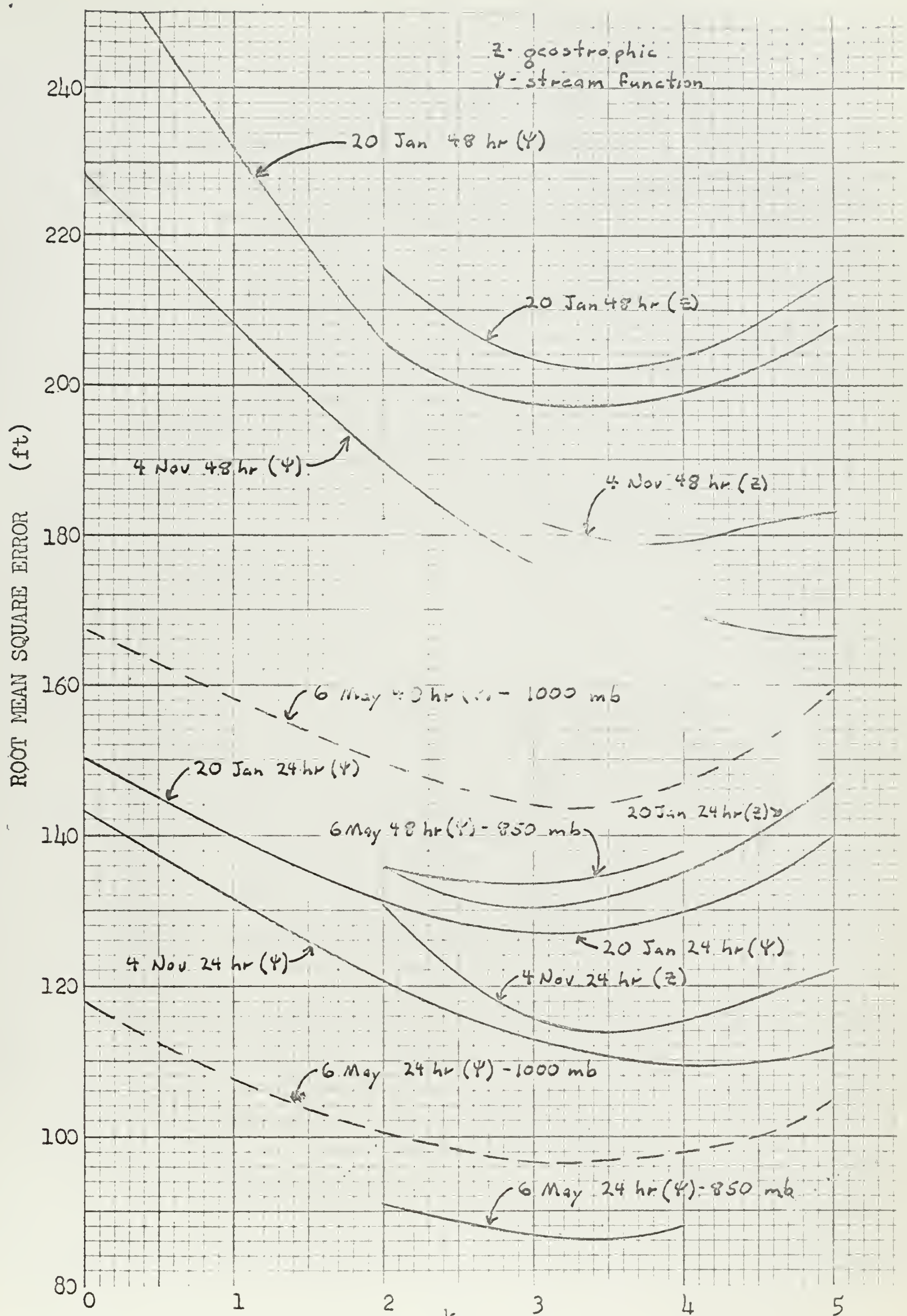


Figure 1. Plot of k_1 vs. RMSE for 850 mb

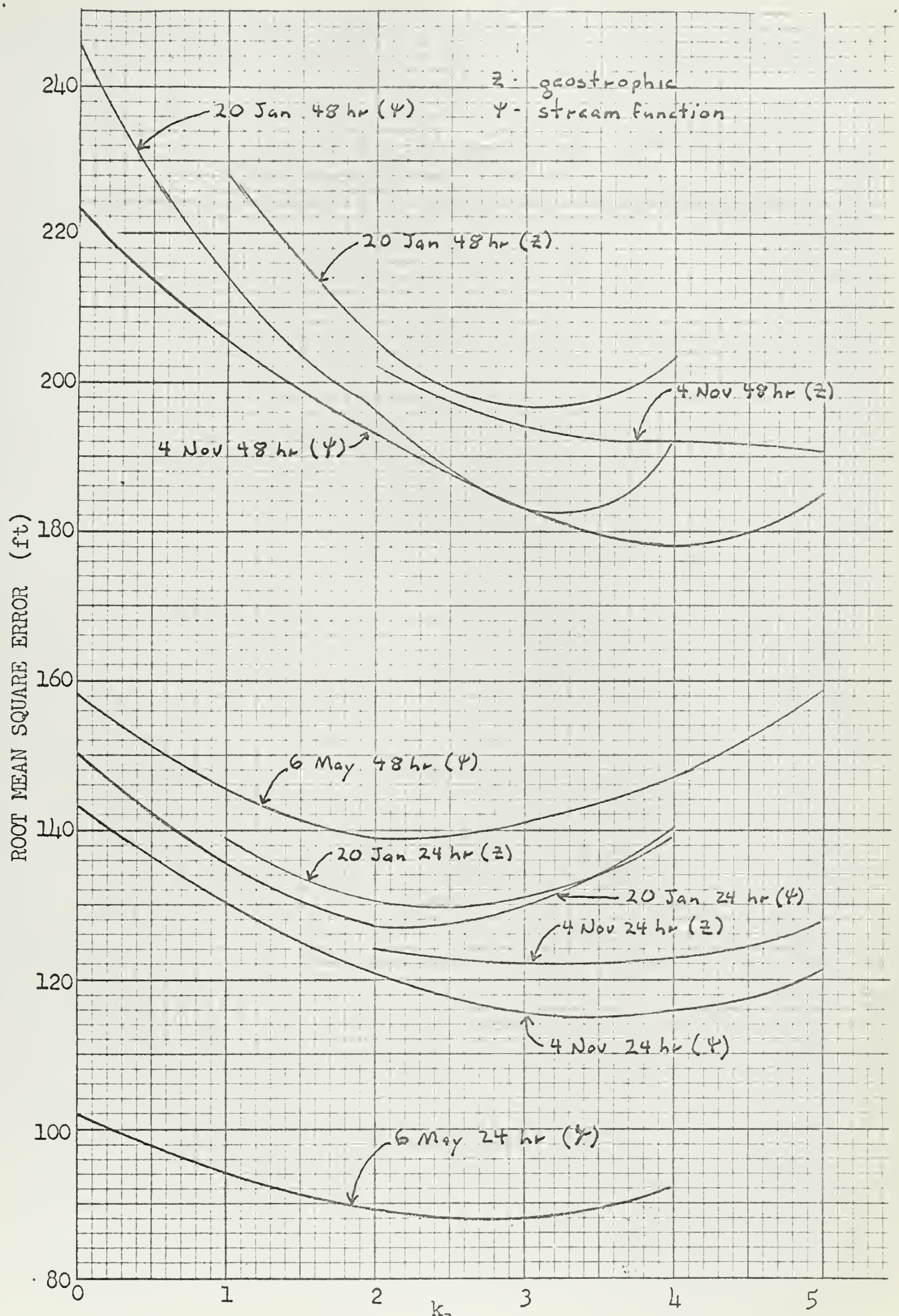


Figure 2. Plot of k_1 vs. RMSE for 700 mb

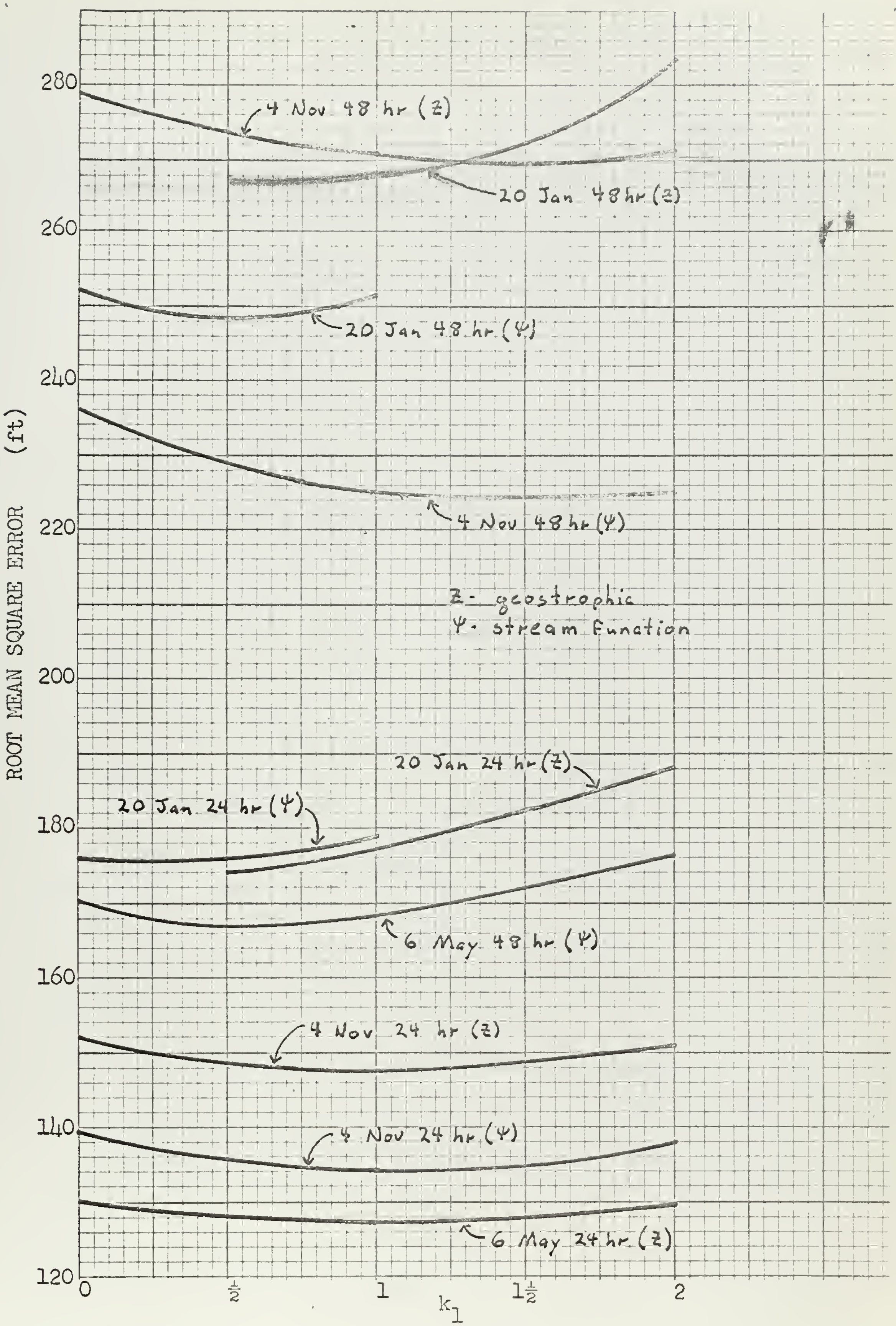


Figure 3. Plot of k_1 vs. RMSE for 500 mb

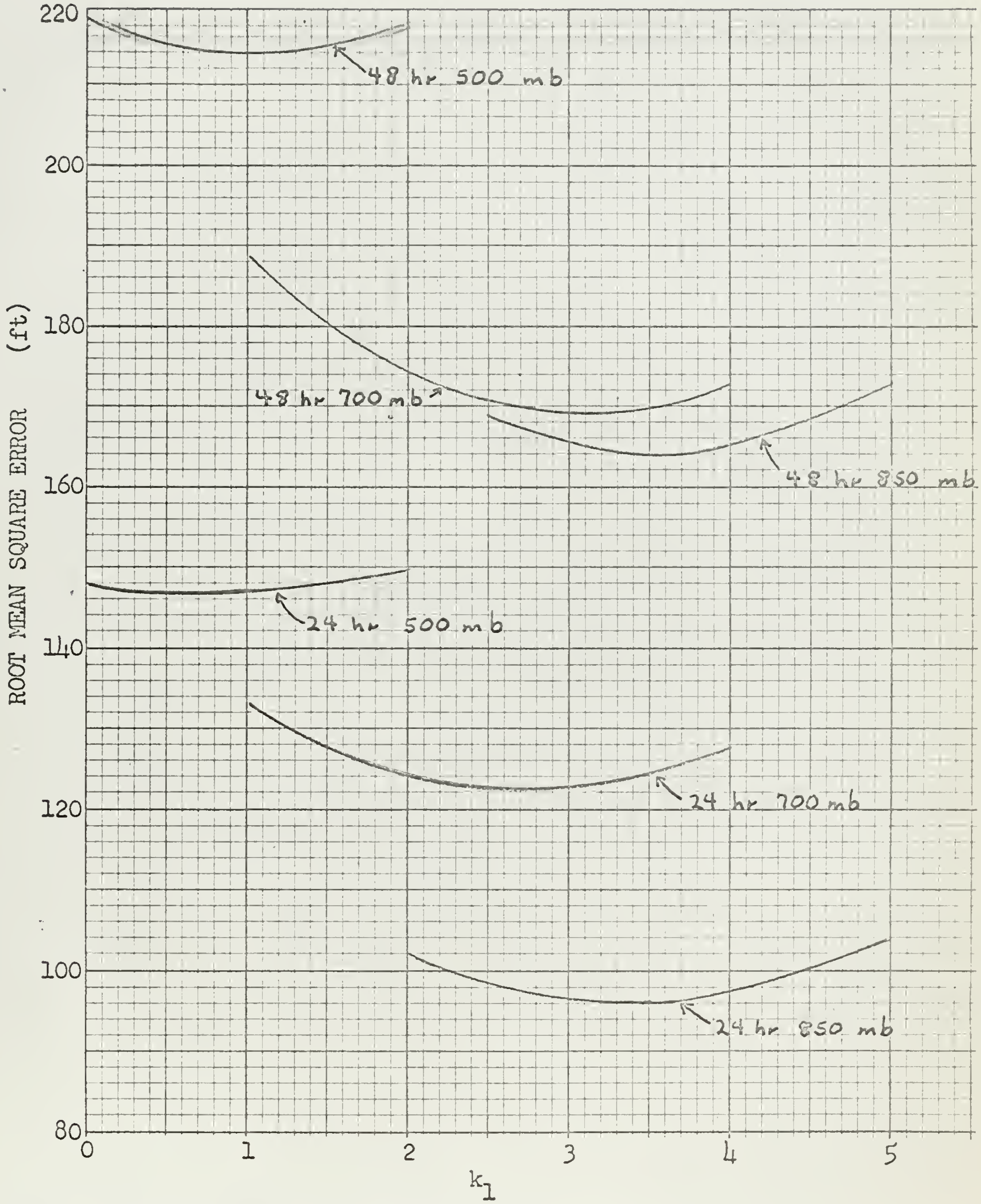


Figure 4. Plot of k_1 vs. mean RMSE

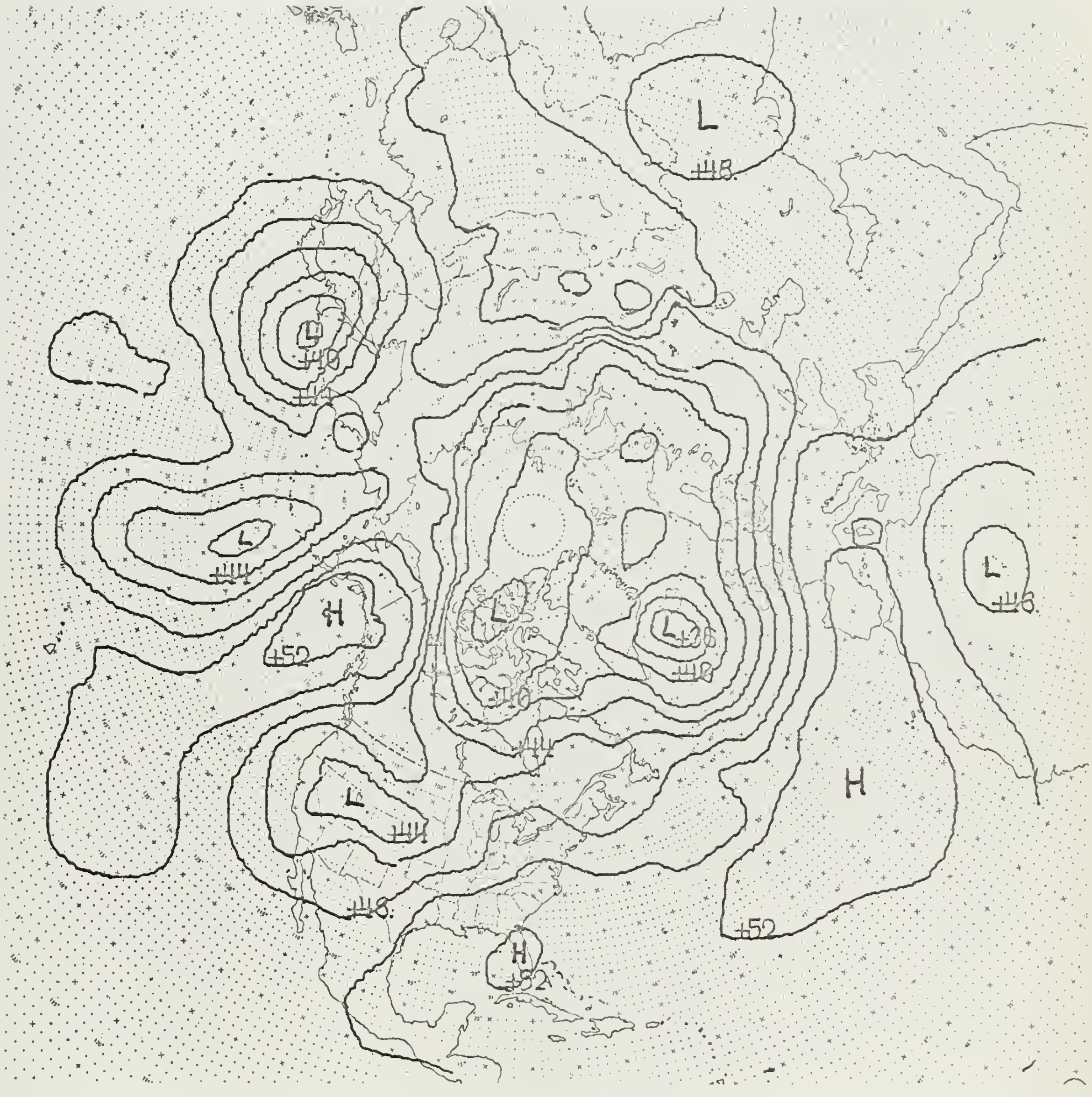


Figure 5. 850-mb analysis, 20 January 1962

Figure 6. 850-mb analysis, 21 January 1962

Figure 7. 850-mb analysis, 22 January 1962

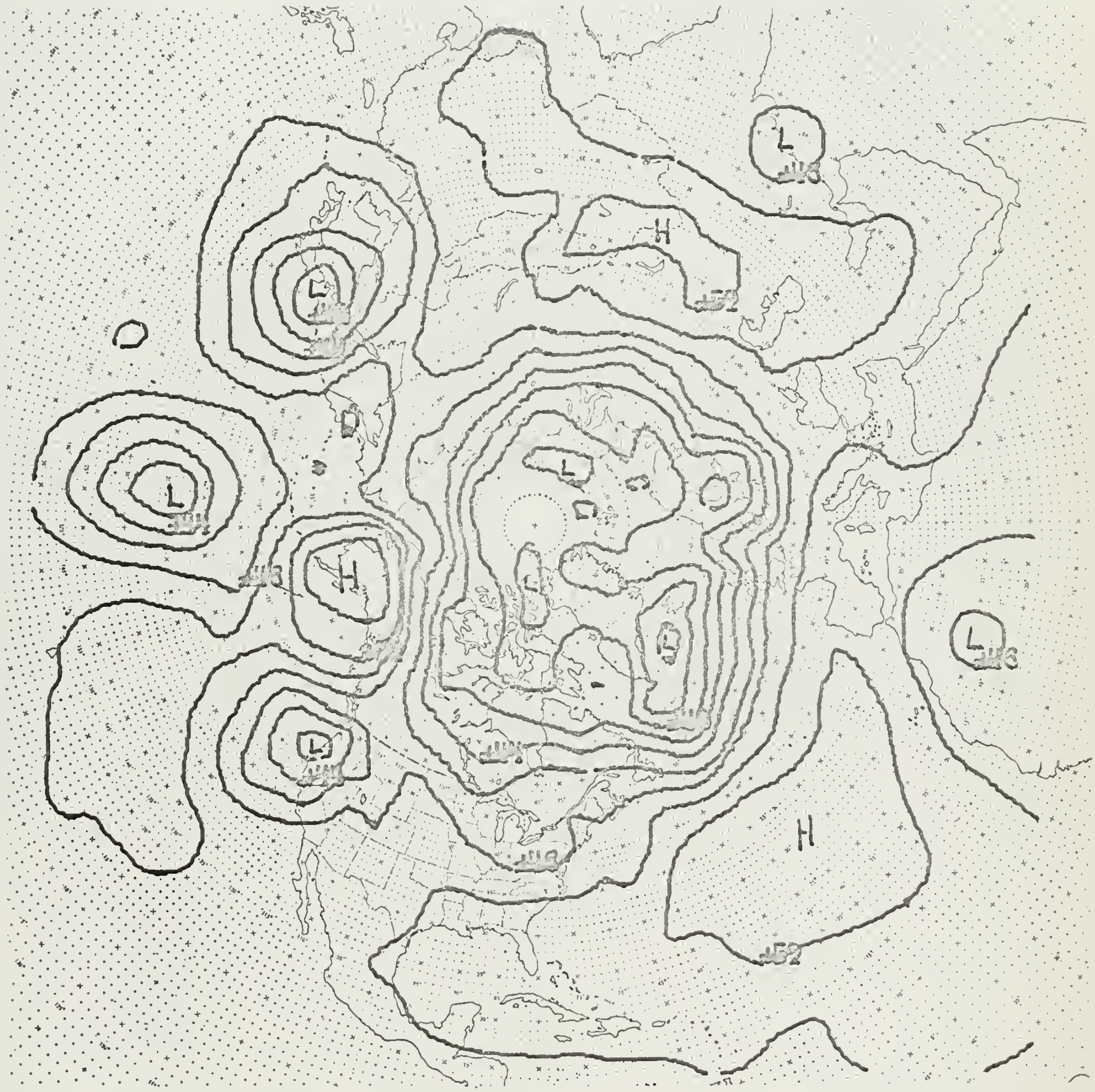


Figure 8. 24-hour forecast for 850 mb from 20 January 1962, k_1=0

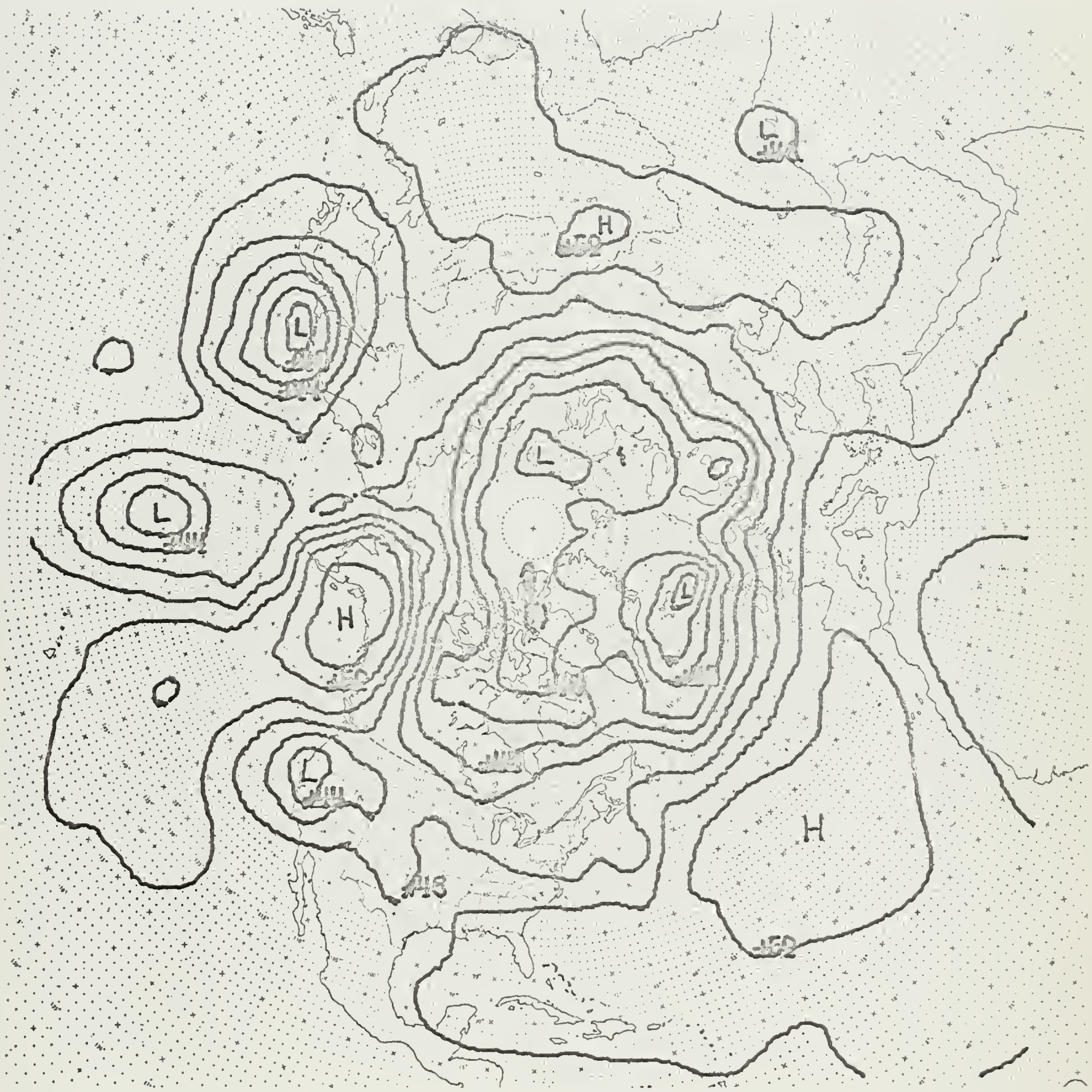


Figure 9. 24-hour forecast for 850 mb from 20 January 1962, $k_1=3$

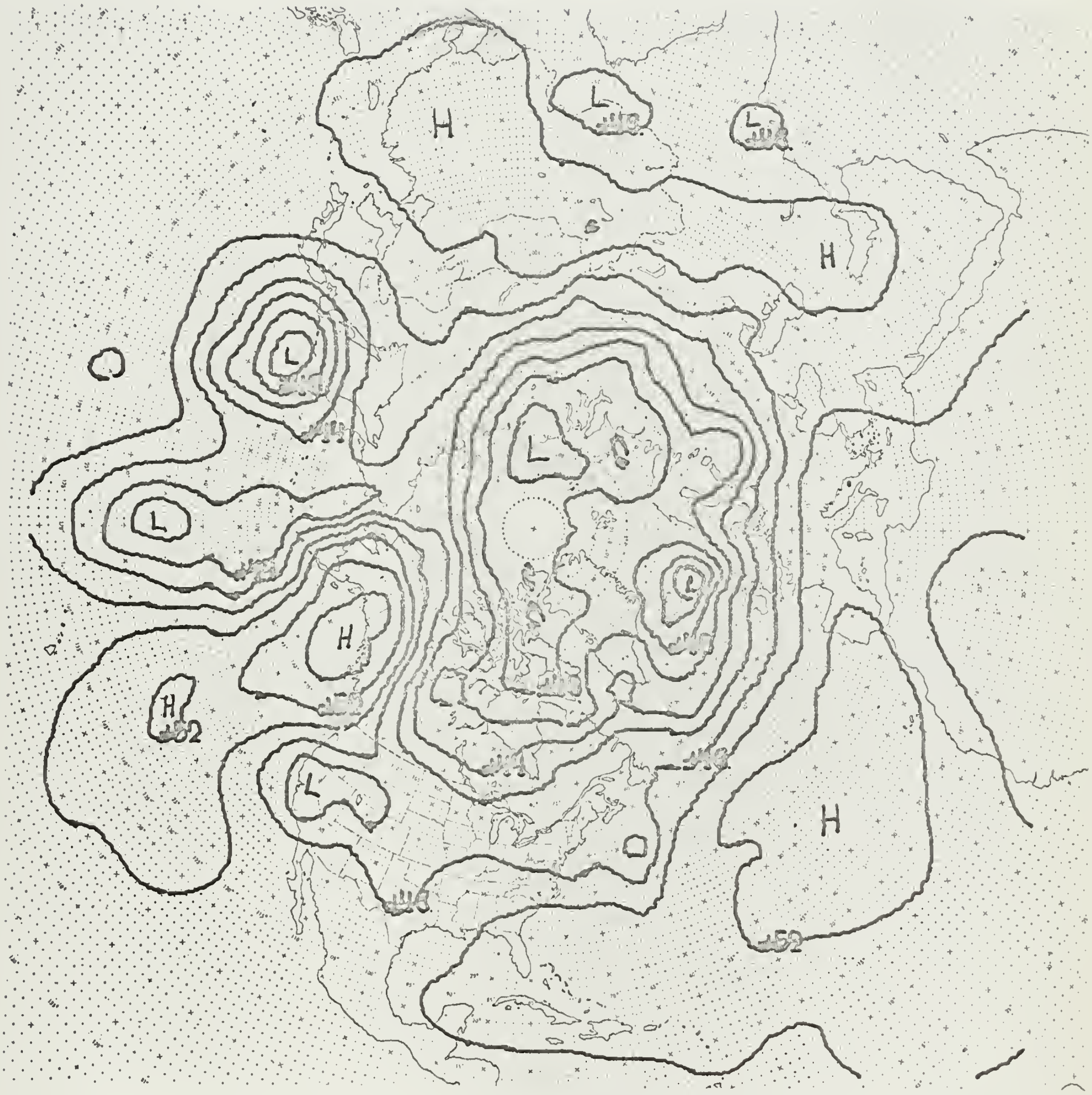


Figure 10. 24-hour forecast for 850 mb from 20 January 1962, $k_1=5$

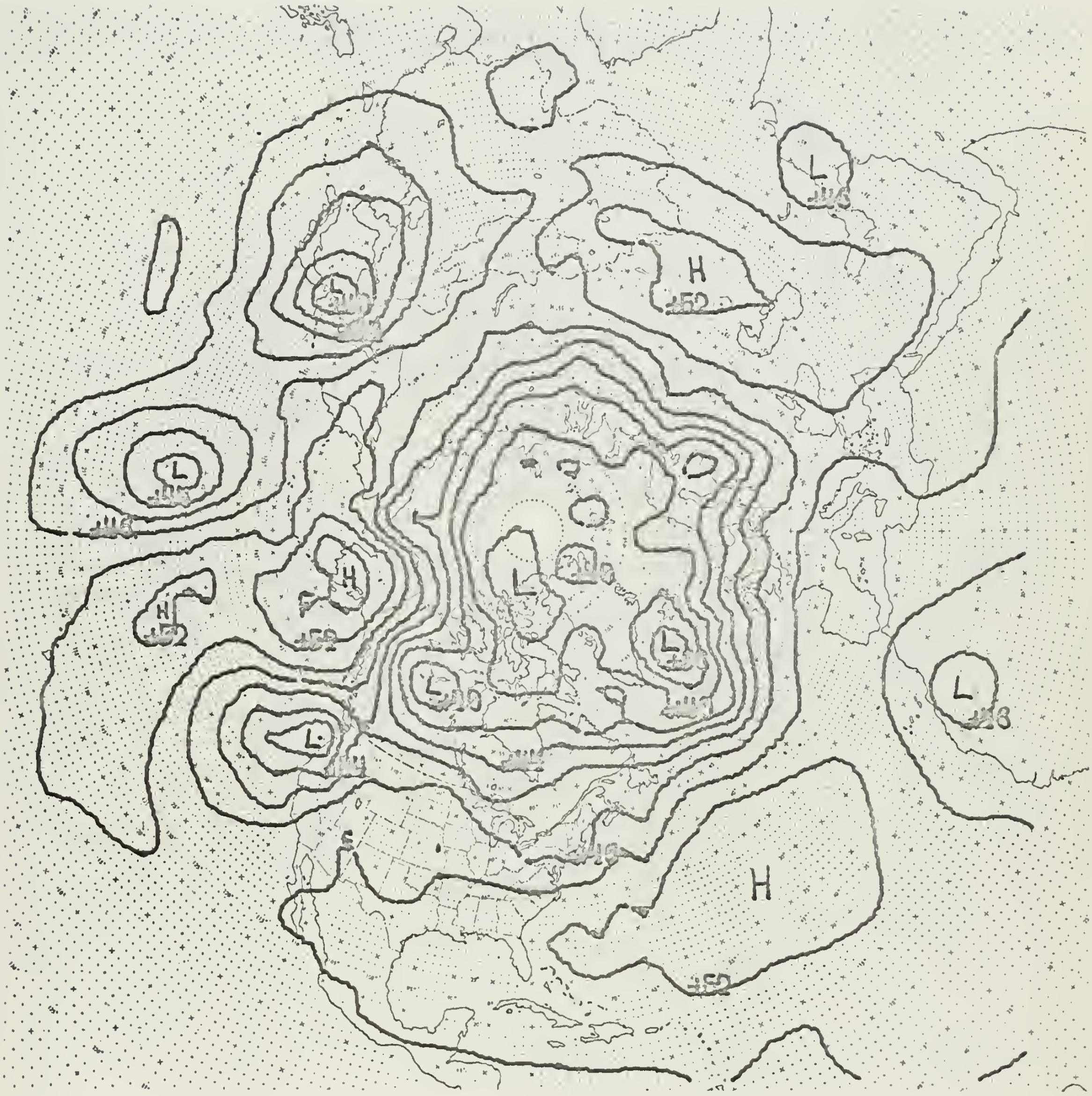


Figure 11. 48-hour forecast for 850 mb from 20 January 1962, $k_1=0$

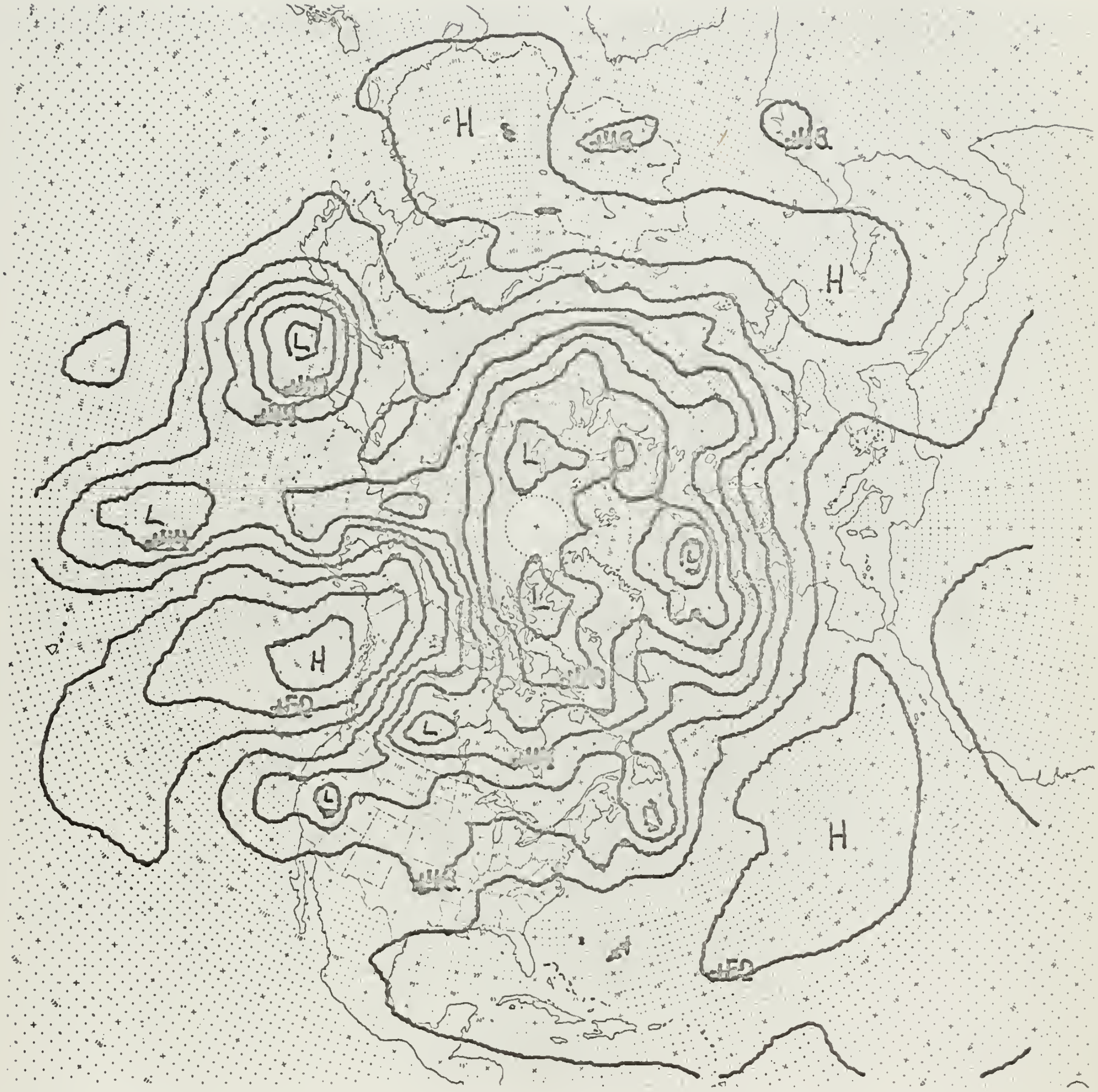


Figure 12. 48-hour forecast for 850 mb from 20 January 1962, k_1=3

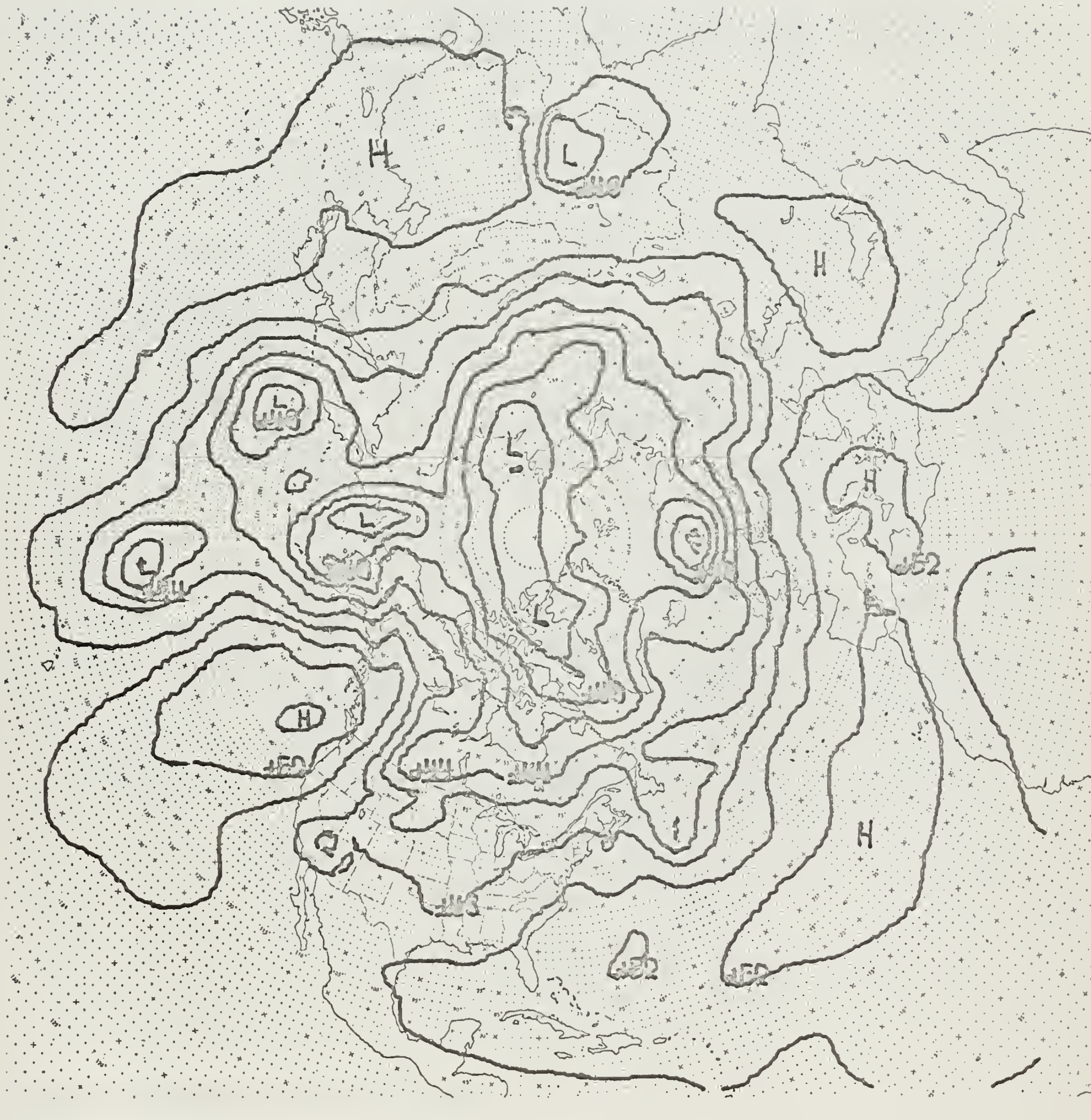
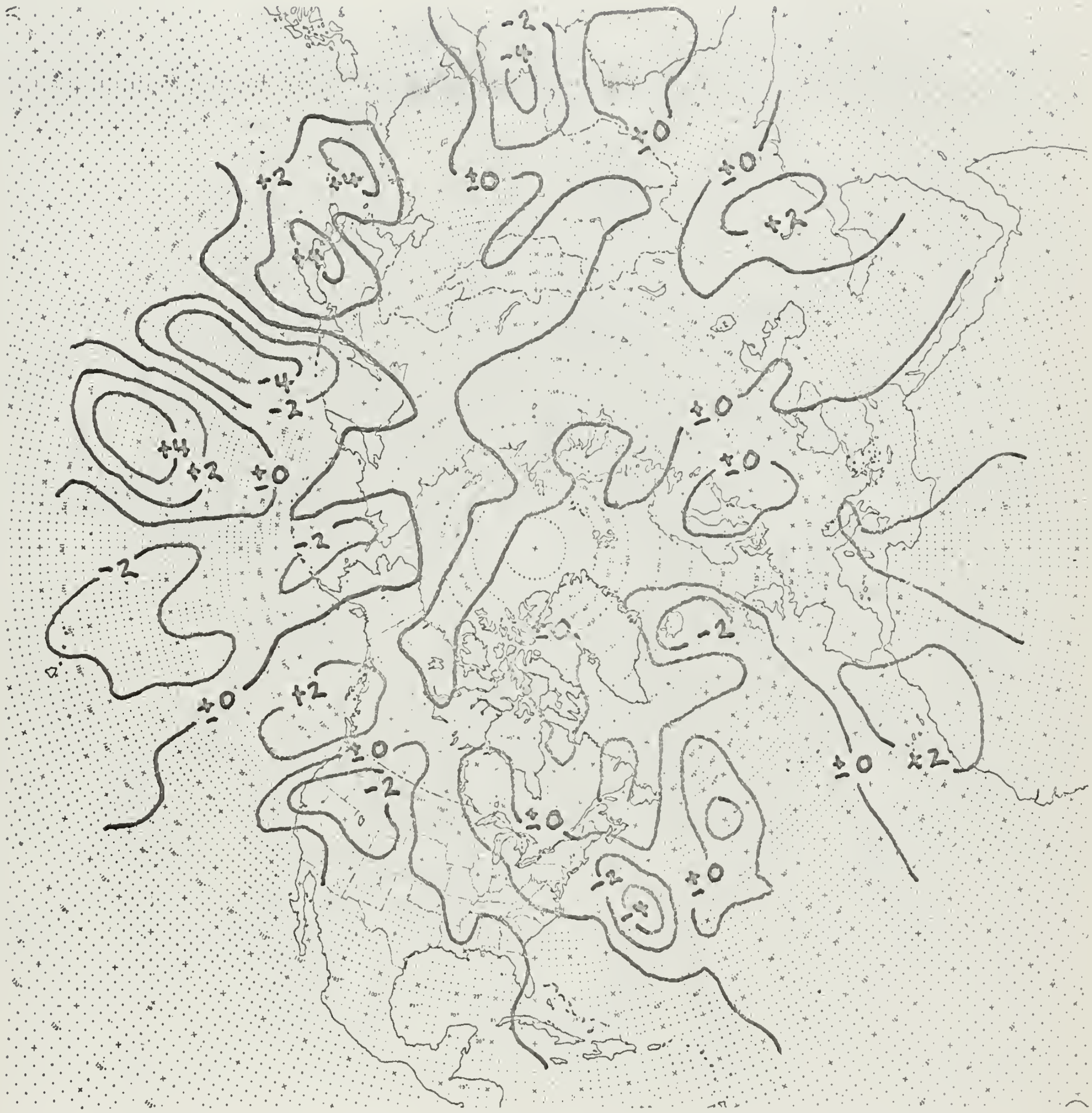
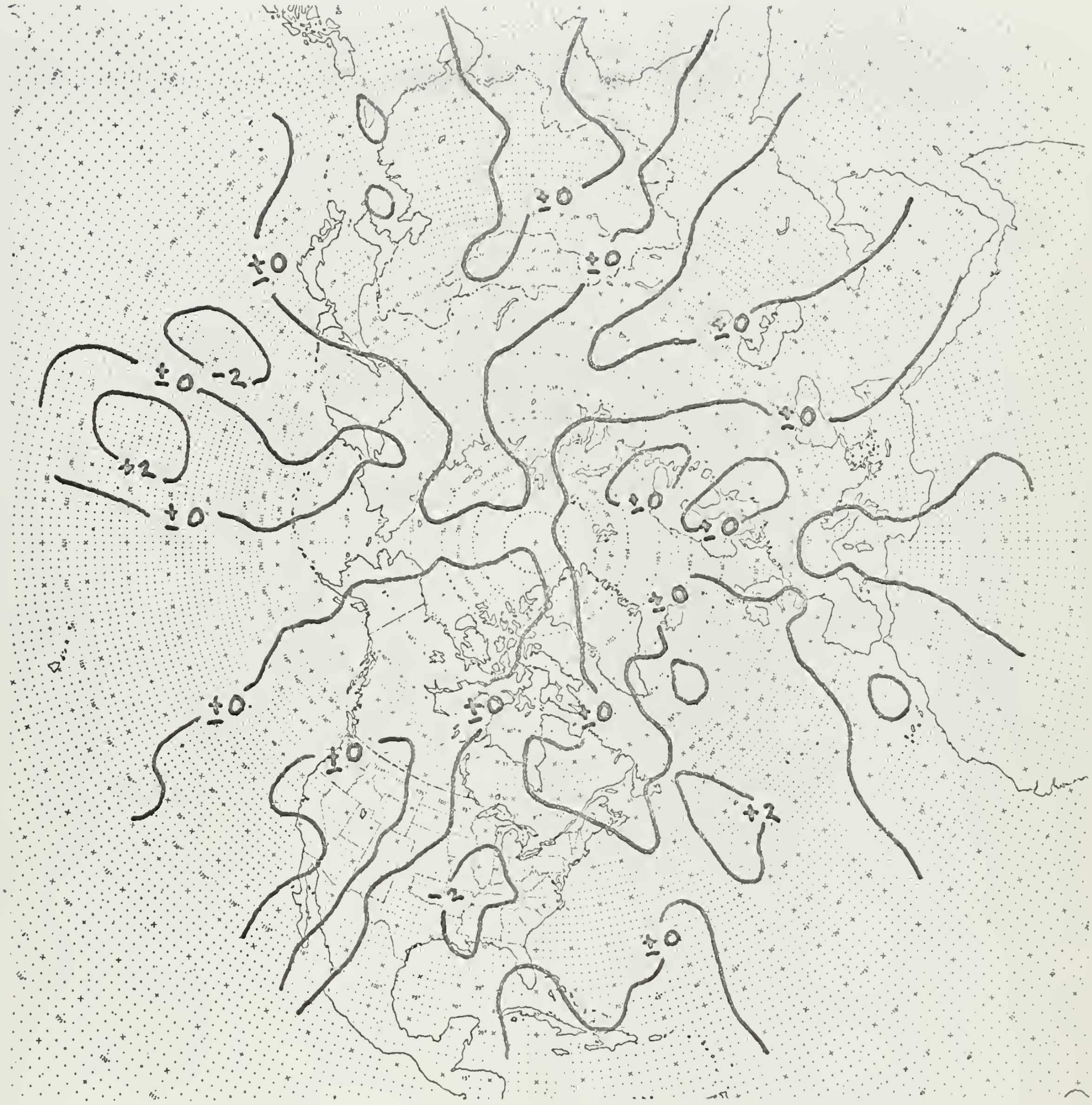


Figure 13. 48-hour forecast for 850 mb from 20 January 1962, k_1=5



contour interval = 2 x 10^{-6} sec^{-1}

Figure 14. 850-mb divergence, 20 January 1962



contour interval = 2 x 10^{-6} sec^{-1}

Figure 15. 700-mb divergence, 20 January 1962

contour interval = 2 x 10^{-6} sec^{-1}

Figure 16. 500-mb divergence, 20 January 1962

Figure 17. 1000-mb analysis, 6 May 1962

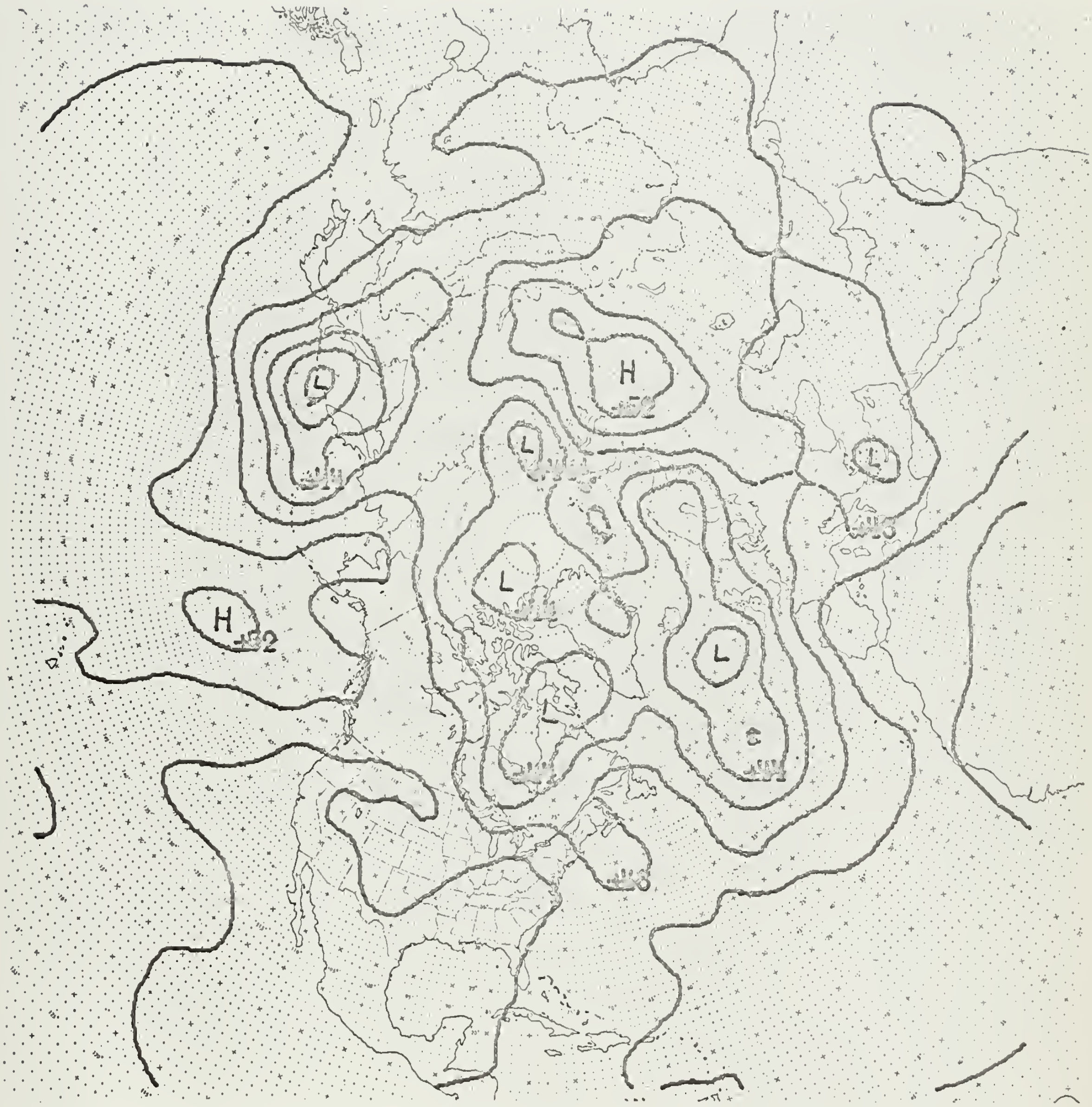


Figure 18. 850-mb analysis, 6 May 1962

Figure 19. 24-hour forecast for 1000 mb from 6 May 1962

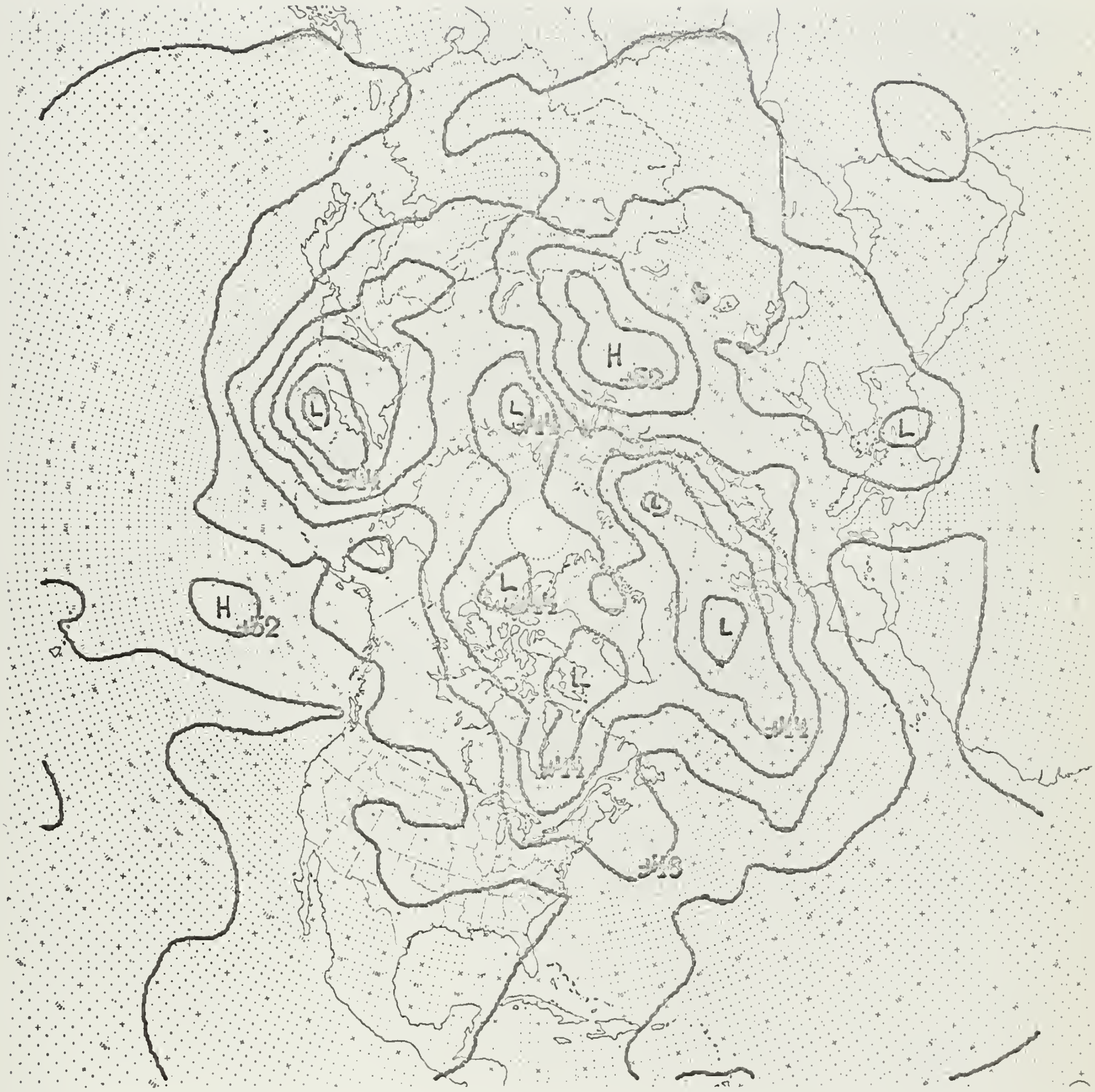


Figure 20. 24-hour forecast for 850 mb from 6 May 1962

Figure 21. 1000-mb analysis, 7 May 1962

Figure 22. 850-mb analysis, 7 May 1962

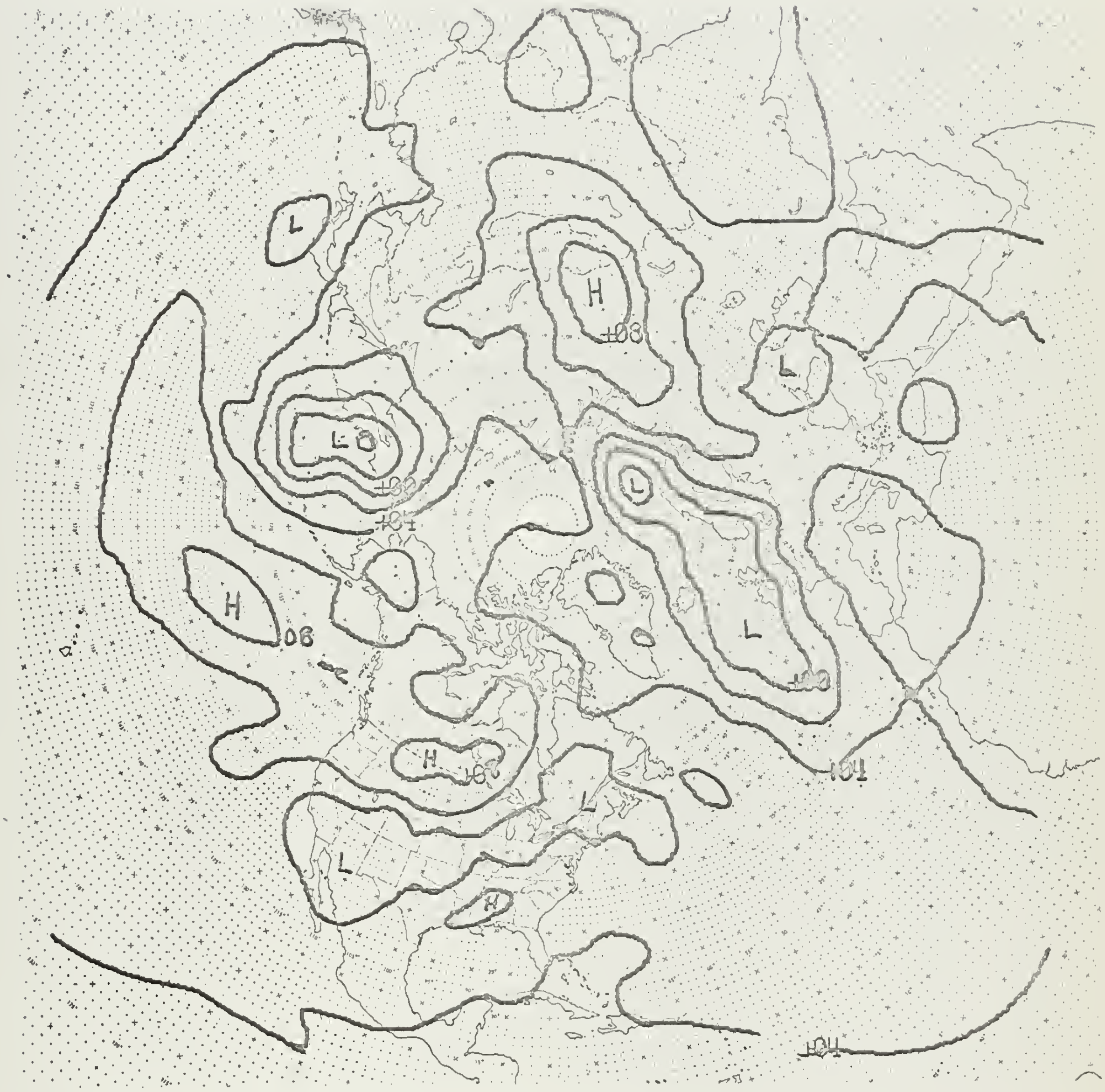


Figure 23. 48-hour forecast for 1000 mb from 6 May 1962

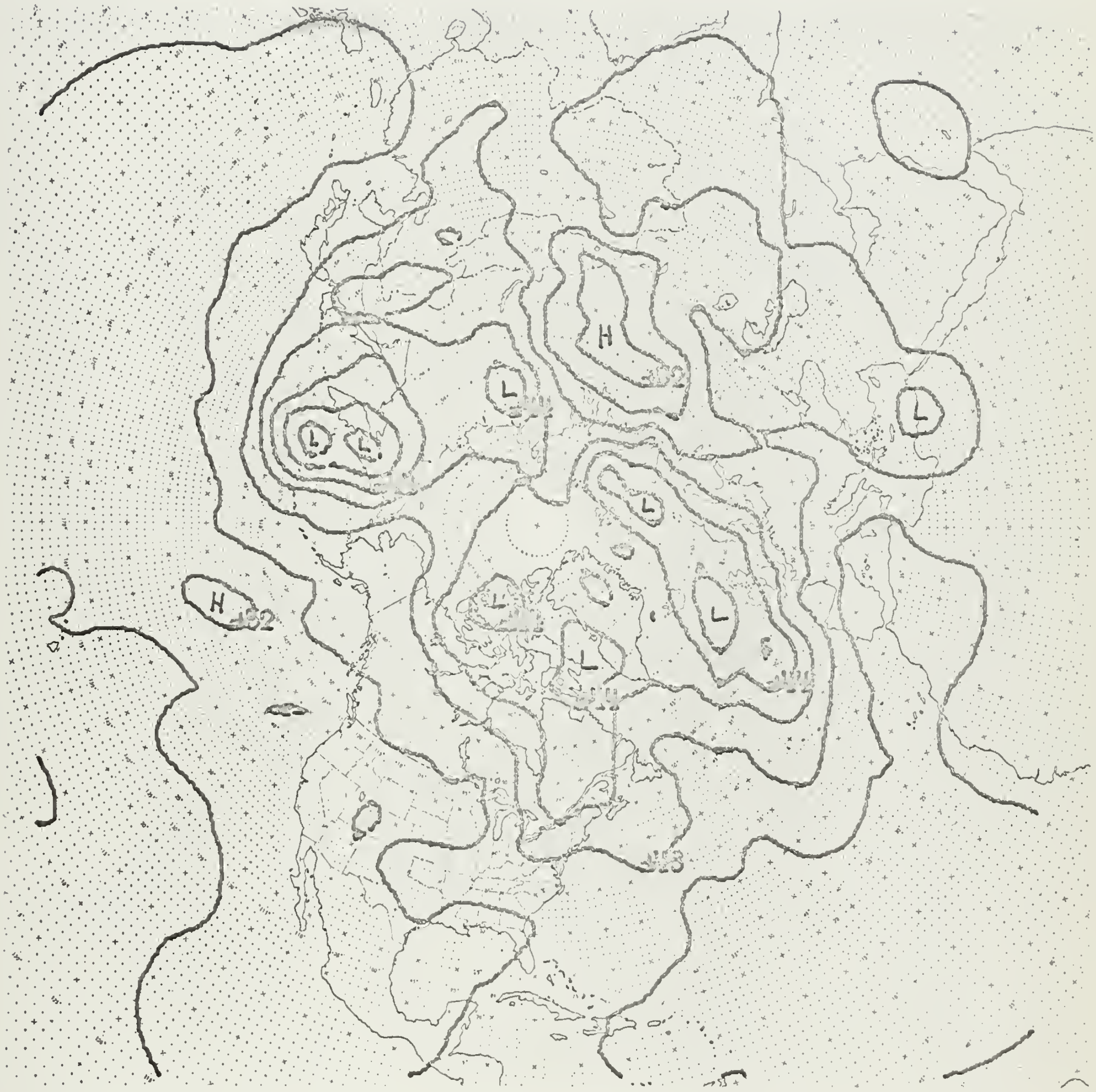


Figure 24. 48-hour forecast for 850 mb from 6 May 1962

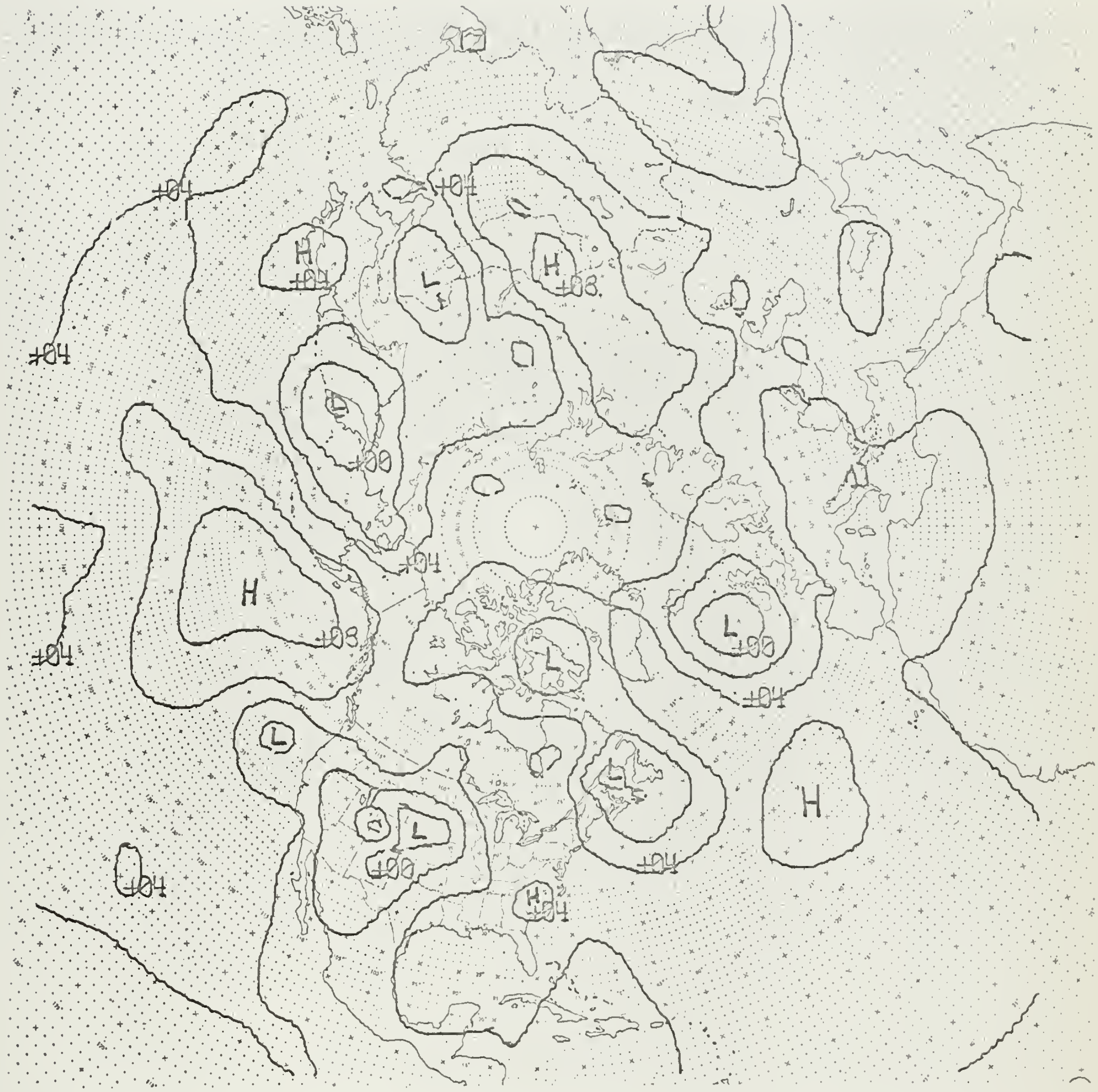


Figure 25. 1000-mb analysis, 8 May 1962

Figure 26. 850-mb analysis, 8 May 1962

Figure 27. 500-mb analysis, 4 November 1961

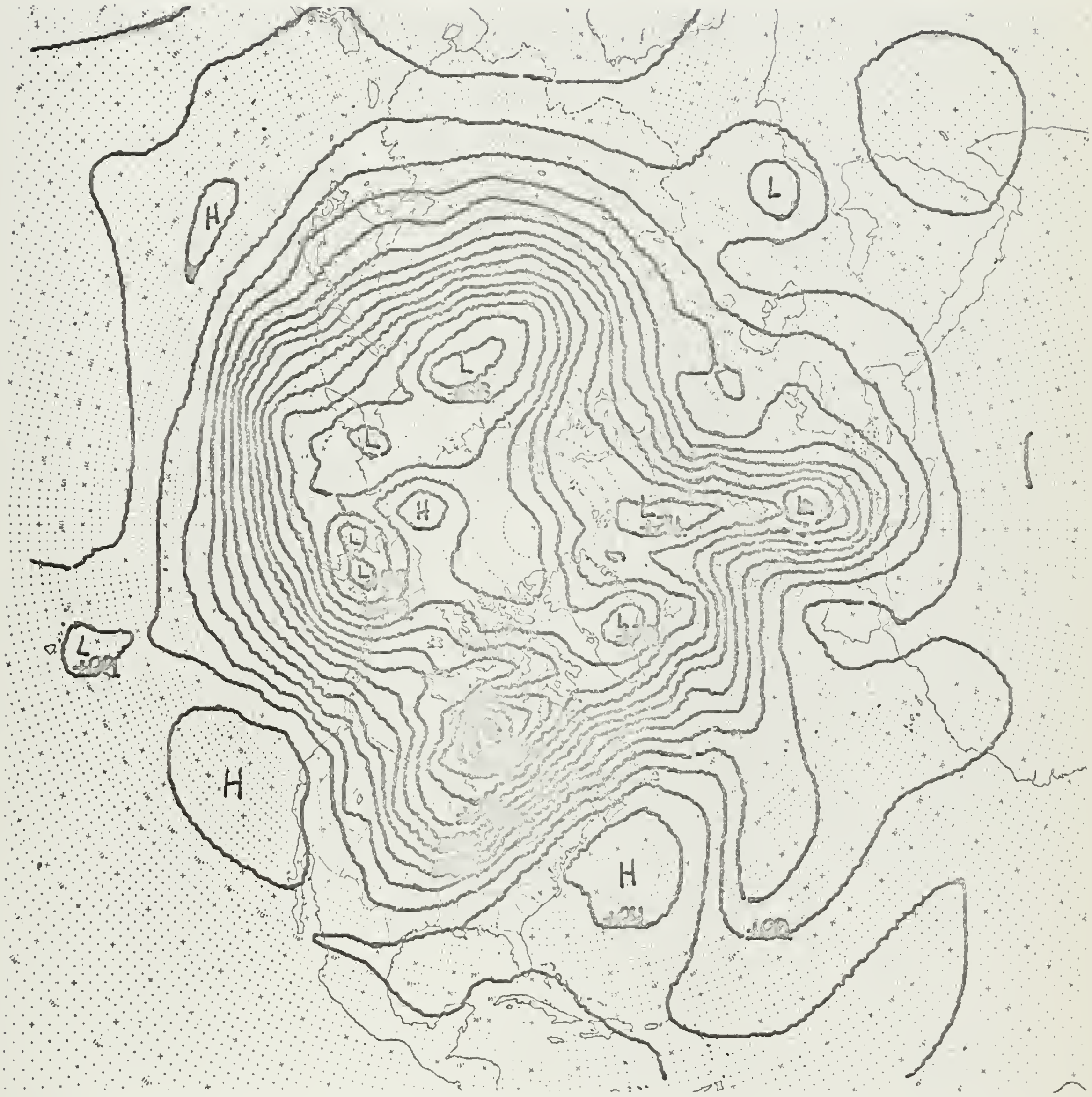


Figure 28. 48-hour forecast for 500 mb from 4 November 1961
(stream function)

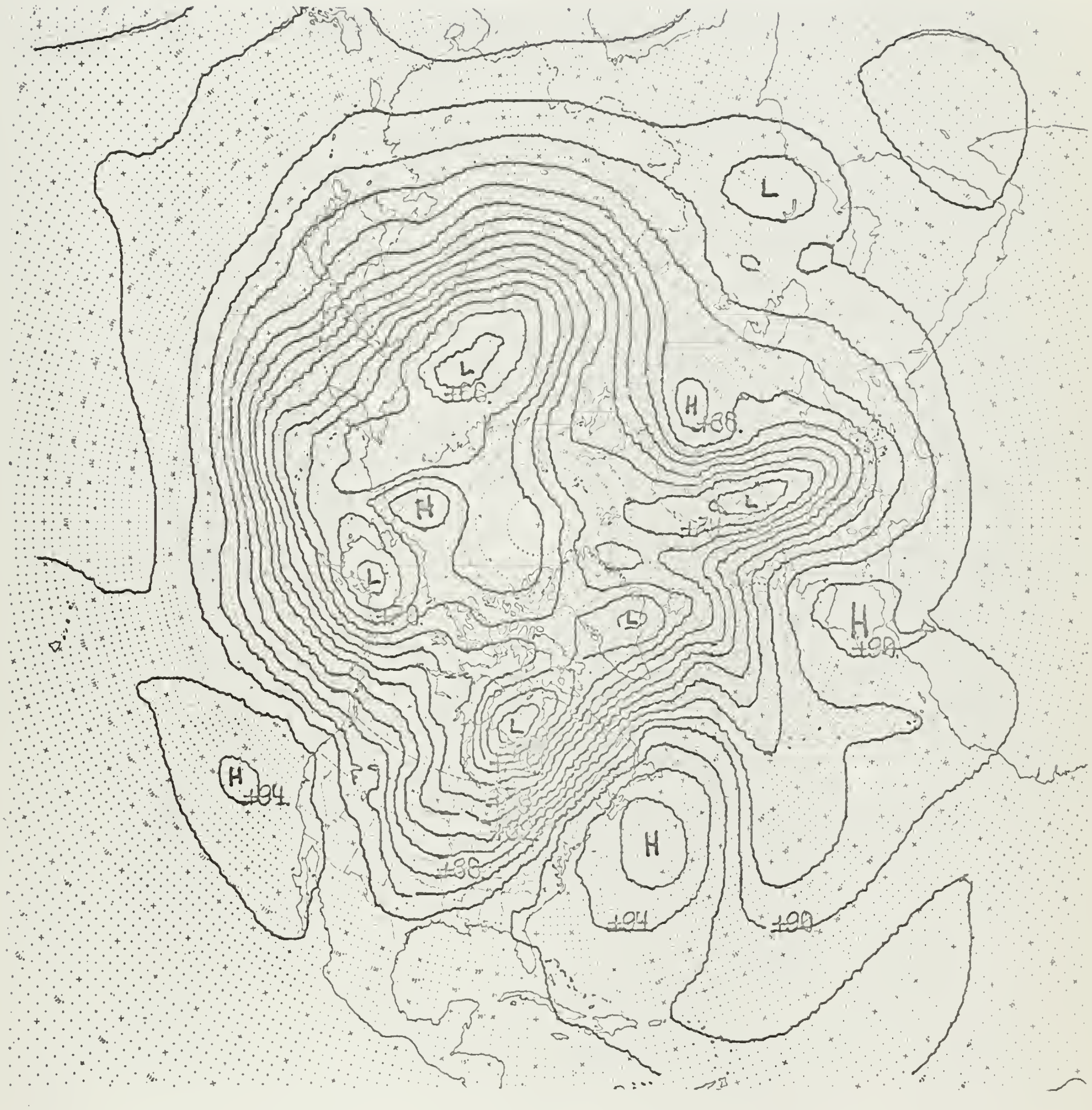


Figure 29. 48-hour forecast for 500 mb from 4 November 1961 (geostrophic)

Figure 30. 500-mb analysis, 6 November 1961

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